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ticularity and concreteness. They are devoided of matter and energy with all their qualities. Thus the determination of a locus is a mere point without extension and its motion produces mere length without breadth or thickness, etc. Everywhere we meet with that subtle fabric of anyness which is a true nothing in the sense of the absence of everything concrete, but not an absolute nothing. In this anyness the mathematician operates and his mode of operation is a work of anyness.

Mathematical space which is the domain of anyness in which the mathematician performs his operations, includes the possibility of constructing even boundaries, and even boundaries are needed for mathematical constructions on account of their quality of being unique. Uniqueness is needed in order to have a standard of reference. The three even boundaries which thus recommend themselves by their uniqueness as standards of reference, are the straight line, the plane, and the right angle, and they make it possible to construct parallel lines. Accordingly it is obvious that the problems of the straight line, of the plane, of the right angle, of the sum of the angles in a triangle as equal to two right angles, and of parallelism are practically the same problem, and it is impossible to construct any one of them from nothing with the help of pure logic only. In addition to pure logic, the mathematician needs for the construction of his science the concept of anyness which yields that most indispensable quality of mathematical space, homogeneity without which mathematics would be impossible.

This idea of anyness is a product of abstraction and the mathematician should know its origin as well as its application in order to understand the foundation of his science.

EDITOR.

SOME REMARKS ON MR. RUSSELL'S ARTICLE, "A MODERN ZENO."

I have been reading with interest the April number of *The Monist,* especially "The Choice of Facts," by H. Poincaré, and "A Newly Discovered Treatise of Archimedes," by J. L. Heiberg. I was attracted by the title "A Modern Zeno," and I was very much surprised to learn the identity of the man. Mr. Russell, the writer of the article, has evidently made some study of Non-Euclidean Geometry, especially of the writings of Lobatchevsky. But truly "a little learning is a dangerous thing." His study has been superficial and without understanding. It is not my intention to criticise the article in detail, but to point out two errors that make it almost worthless.

On page 294, it is stated that the straight line containing the vertex of an isosceles right triangle and the midpoint of the hypotenuse divides the triangle into two equal isosceles right triangles. That these two triangles are equal right triangles is true in the geometry of Lobatchevsky, but that they are *isosceles* cannot be proved from his assumptions, although Mr. Russell says that the proof is so simple that it would be an imputation upon the reader to spread it before him.

On pages 300-302, Mr. Russell has given what he thinks is a proof that the geometry of Lobatchevsky is self-contradictory. His error is due to the fact that he used Theorem 25 without understanding it. It is true that he has stated it in the exact words of Halsted's translation of Lobatchevsky's *Researches on the Theory of Parallels,* namely "Two straight lines which are parallel to a third are parallel to each other"; but either he did not read the proof given there or he did not understand it. The theorem as stated is incorrect or perhaps I should say it is incomplete. But this fact would not have been misleading if he had read and understood the proof. On page 34 of H. P. Manning's *Non-Euclidean Geometry,* the theorem is more carefully stated as follows: "Two lines parallel to a third toward the same part of the third are parallel to each other." Indeed it is stated on page 13 of Halsted's translation that in the geometry of Lobatchevsky we must make a distinction of *sides in parallelism.* Mr. Russell's failure to take account of this distinction vitiates his argument. That he has utterly failed to comprehend the distinction is evidenced by the following statement taken from the bottom of page 302: "Now by the very same course of deduction (no step of which is unsanctioned in the 'system' of Lobatchevsky) the line KAK′ is shown parallel to HAH′ and to EAE′, in spite of the rather important fact that they cut one another at A."

It seems that Mr. Russell has some doubt as to the correctness of his conclusion, for on page 303 are these words: "Still it may be that there is something about the matter that I do not understand. If so, I can only protest that my failure is not due to any lack of respectful (I do not want to say absurdly respectful) study of Lobachevsky's little brochure." It looks to me as if Mr. Russell did want to say "absurdly respectful." My comment is this: Perhaps it was respectful. Certainly it was superficial and without much comprehension, especially in connection with Theorem 25. The first of the two errors I have mentioned shows his lack of understanding of the details of Lobatchevsky's geometry, but it is not so serious because it led him merely to some remarks about a "bent plane." The second error is more serious because it led him to the conclusion that Lobatchevsky's geometry is self-contradictory and that "we find Lobatchevsky hitting upon the right and sufficient way of proving the parallel postulate of Euclid."

The Monist is devoted to the philosophy of science, and articles on Non-Euclidean Geometry are certainly not out of place in its pages. A good paper on the subject or its philosophic import may be written by one who is not an expert mathematician, but it seems to me that such a paper should be carefully read by an expert mathematician before publication, so that errors due to the author's lack of knowledge of the technique of the mathematics involved may be eliminated. Certainly this should be done when an author thinks he has found a fallacy in a doctrine accepted as sound by mathematicians the world over.

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PROFESSOR LOVEJOY ON "DER VORCHRISTLICHE JESUS."

The following reply to Professor Lovejoy's criticism was promised in a footnote for the January number of *The Monist* and was in fact written out in full nearly two weeks before October 29, 1908, when I sailed for Chile. But as the protracted absence from the United States that followed rendered it quite impossible either to give the paper final revision or for me to see the proofs, if it should be issued in January, it seemed best to hold it back for the present number. The occasion for any rejoinder whatever is supplied not by the argumentative appeal of the review, which may be safely left to the judgment of readers of the book, but by its strictures upon the author's treatment of authorities, especially of Hippolytus.

Imprimis, let me thank Professor Lovejoy for the general manner of his review. While not exposing fully the argumentative nerve of the work in hand, he seems really to have intended to get at the heart of the matter, and his statement of the main drift of the essays calls for acknowledgement. Moreover, he has not shrunk